Math 209 page 19 #79
Simplify:

$$f(x)=\sqrt{x}$$

$$\frac{f(a+h)-f(a)}{h}=\frac{\sqrt{a+h}-\sqrt{a}}{h}$$

Solution:

$$\begin{aligned}
\frac{f(a+h)-f(a)}{h} &= \frac{\sqrt{a+h}-\sqrt{a}}{h} \\
&= \frac{\sqrt{a+h}-\sqrt{a}}{h}\cdot\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}} \\
&= \frac{\left(\sqrt{a+h}\right)^2-\left(\sqrt{a}\right)^2}{h}\cdot\frac{1}{\sqrt{a+h}+\sqrt{a}} \\
&= \frac{a+h-a}{h}\cdot\frac{1}{\sqrt{a+h}+\sqrt{a}} \\
&= \frac{h}{h}\cdot\frac{1}{\sqrt{a+h}+\sqrt{a}} \\
&= 1\cdot\frac{1}{\sqrt{a+h}+\sqrt{a}} \\
&= \frac{1}{\sqrt{a+h}+\sqrt{a}}
\end{aligned}$$