Math 209 page 19 #80
Simplify:

$$f(x)=\frac{1}{x}$$

$$\frac{f(a+h)-f(a)}{h}$$

Solution:

$$\frac{f(a+h)-f(a)}{h}=\frac{\frac{1}{a+h}-\frac{1}{a}}{h}$$

$$=\frac{\frac{1}{a+h}\cdot\frac{a}{a}-\frac{1}{a}\cdot\frac{a+h}{a+h}}{h}$$

$$=\frac{\frac{a}{a(a+h)}-\frac{a+h}{a(a+h)}}{h}$$

$$=\frac{\frac{a-(a+h)}{a(a+h)}}{h}$$

$$=\frac{\frac{a-a-h}{a(a+h)}}{h}$$

$$=\frac{\frac{-h}{a(a+h)}}{h}$$

$$=\frac{-h}{a(a+h)}\div h$$

$$=\frac{-h}{a(a+h)}\cdot\frac{1}{h}$$

$$=\frac{-1}{a(a+h)}$$